Laplace & Fourier Transform Formulas

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Quick Definition: The unilateral Laplace transform is defined as $F(s) = \mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t) \, dt$ for $t \ge 0$, where $s = \sigma + j\omega$ is a complex frequency variable.

1. Common Laplace Transform Pairs

Time Domain $f(t)$	Laplace Domain $F(s)$	Description / Notes
$\delta(t)$	1	Unit Impulse Function
$u(t)$ (or 1)	$\frac{1}{s}$	Unit Step Function ($\sigma > 0$)
t	$\frac{1}{s^2}$	Linear Ramp Function
t^n	$\frac{n!}{s^{n+1}}$	Integer Power ($n = 0, 1, 2, \dots$)
e^{-at}	$\frac{1}{s + a}$	Exponential Decay
$t \cdot e^{-at}$	$\frac{1}{(s + a)^2}$	Shifted Ramp
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$	Sine Waveform
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$	Cosine Waveform
$e^{-at} \sin(\omega t)$	$\frac{\omega}{(s + a)^2 + \omega^2}$	Damped Sine Wave
$e^{-at} \cos(\omega t)$	$\frac{s + a}{(s + a)^2 + \omega^2}$	Damped Cosine Wave

2. Essential Laplace Operational Properties

Property	Time Domain $f(t)$	Laplace Domain $F(s)$
Linearity	$a f_1(t) + b f_2(t)$	$a F_1(s) + b F_2(s)$
Time Shifting	$f(t - t_0) u(t - t_0)$	$e^{-s t_0} F(s)$
Frequency Shifting	$e^{-at} f(t)$	$F(s + a)$
Time Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
First Derivative	$\frac{d}{dt} f(t)$	$s F(s) - f(0^-)$
Second Derivative	$\frac{d^2}{dt^2} f(t)$	$s^2 F(s) - s f(0^-) - f'(0^-)$

3. Advanced Laplace Theorems

Theorem	Expression	Application / Notes
Integration in Time	$\int_{0}^{t} f(\tau) \, d\tau$	$\frac{1}{s} F(s)$
Multiplication by t	$t \cdot f(t)$	$-\frac{d}{ds} F(s)$
Division by t	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(u) \, du$
Initial Value Theorem	$f(0^+)$	$\lim_{s \to \infty} s F(s)$
Final Value Theorem	$\lim_{t \to \infty} f(t)$	$\lim_{s \to 0} s F(s)$ (if poles are in LHP)

4. Fourier Transform Pairs & Properties

The continuous Fourier transform connects time-domain signals directly to the frequency spectrum $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$.

Time Domain $x(t)$	Frequency Domain $X(\omega)$	Remarks
$\delta(t)$	1	Constant spectral density
1 (Constant)	$2\pi \delta(\omega)$	DC component
$e^{-a\|t\|}$ ($a > 0$)	$\frac{2a}{a^2 + \omega^2}$	Double-sided exponential
$\text{rect}(t)$	$\text{sinc}\left(\frac{\omega}{2\pi}\right)$	Gate / Rectangular pulse
$\cos(\omega_0 t)$	$\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	Pure sinusoidal tone
$x(t - t_0)$	$X(\omega) e^{-j\omega t_0}$	Time Shifting Property
$\frac{d}{dt} x(t)$	$j\omega X(\omega)$	Differentiation Property

Study Tip for Engineers: Keep this cheat sheet handy during control systems and circuit analysis homework. Always check region of convergence (ROC) stability criteria when applying inverse Laplace transforms via partial fraction expansion.